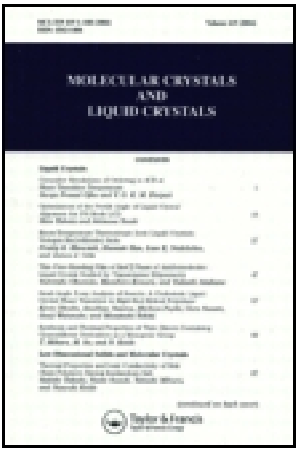

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Optical Surface Guided Waves in Chiral Liquid Crystal Films

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Excitation of Surface guided electromagnetic waves (SGEW) of optical wave range in chiral liquid crystals (CLC) are theoretically examined in the framework of dynamical diffraction theory. In contrast to the traditionally discussed method of SGEW excitation by means of attenuated total external reflection (ATER) the SGEW excitation by a beam incident at a CLC film from the side of a boundary without total internal reflection (TIR) is investigated. In the general case an analytical description of the SEW excitation can not be carried out. It is why the limiting case allowing simplification of the problem, namely, SGEW of the second order diffraction, is examined in details.

INTRODUCTION

Excitation of SGEW by means of ATER for semi-infinite CLC was examined in [1]. For CLC films there is another possibility to excite SGEW by light incident at the film from the interface where TIR is absent. In this case incident light experiences diffraction in a CLC film and excites SGEW if the structure of diffracting wave is similar to the SGEW field structure. A general consideration of the problem was performed in [2]. It was shown that the efficiency of SGEW excitation is strongly dependent on the propagation direction of SGEW relative to the director orientation at the CLC surface, polarization of the incident beam, film thickness and reaches a maximum value for the definite, dependent on light frequency, beam incidence angle.

The SGEW field structure is determined by a solution of the Maxwell equations with the corresponding boundary conditions. Due to the smallness of the CLC dielectric anisotropy δ for the solution of Maxwell equation in CLC film the results of CLC optics in the two-wave dynamical diffraction approximation [3] may be used with good accuracy. Thus a general approach to the problem looks as the following.

Let's consider excitation of SGEW in a CLC film of planar texture surrounded by isotropic homogeneous media with dielectric constant ε_1 and ε_3 (see Fig. 1). The spatially periodic dielectric tensor of CLC film is assumed to be of the conventional form [3], with $\bar{\varepsilon} = 1/2(\varepsilon_x + \varepsilon_y)$, $\delta = (\varepsilon_x - \varepsilon_y)/(\varepsilon_x + \varepsilon_y)$; ε_x, ε_y are the principal values of the CLC dielectric tensor, z-axis is directed along the cholesteric axis, ψ is the angle between x-axis and the director at the CLC surface ($z = 0$), τ is the CLC reciprocal lattice vector. It will be assumed that the total internal reflection definitely takes place at the upper boundary ($z = 0$) i.e. $\varepsilon_1 < \varepsilon_x, \varepsilon_y$.

Supposing that SGEW of frequency ω propagates along the x-axis the solution of Maxwell equations will be sought in the form $E_j(r, t) = E_j(z) e^{i(qx - wt)}$ where the index $i = 1, 3$ corresponds to the quantities in the homogeneous media and $i = 2$ corresponds to the quantities in CLC film. The factor $e^{i(qx - wt)}$ will be omitted below. Thus the expressions for the fields take the following form:

$$E_1(Z) = \left(A\hat{y} + B\left(\hat{x} + \frac{iq}{\gamma_1}\hat{z}\right)\right] e^{-\gamma_1 z}, \quad E_3(z) = \left[D\hat{y} + \mathscr{E}\left(\hat{x} + \frac{iq}{\gamma_3}\hat{z}\right)\right] e^{\gamma_3 z}$$

$$E_2(z) = \sum_{j=1}^{4} C_j e^{-\gamma_{2j} z} [\hat{y}(a_{1j} e^{-i\tau z/2} + a_{3j} e^{i\tau z/2}) + \hat{x}\sin\theta \cdot \tag{1}$$

$$\cdot (a_{2j} \bar{e}^{i\tau z/2} - a_{4j} e^{i\tau z/2}) + \hat{z}\cos\theta (a_{2j} e^{-i\tau z/2} + a_{4j} e^{i\tau z/2})],$$

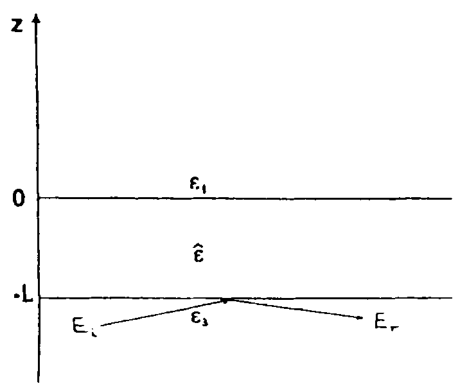

FIGURE 1 Scheme of SGEW excitation in CLC by a beam incident on an interface without TIR.

where $\hat{x}, \hat{y}, \hat{z}$ are the unit vectors along corresponding vectors, $\gamma_{1,3} = [q^2 - (w/c)^{2^{1/2}} \varepsilon_{1,2}]$, A, B, C_j, D, and $\mathscr{E}$ are the coefficients to be determined from the boundary conditions, γ_{2j}, a_{ij} are the imaginary components of corrections to the wave vectors and the parameters determining eigen solutions found in the two-wave approximation of the dynamical diffraction theory (the explicit form of γ_{2j} and a_{ij} are given in [3]), $t_g \theta = \tau/2q$.

One finds the dispersion equation for SGEW from the conventional boundary conditions. The corresponding conditions result in a system of eight linear equations for the eight coefficients A, B, C_j, D. Equating to zero the determinant of this system one gets the dispersion equation for SGEW in CLC film:

$$\det|G_{ik}| = 0 \quad 1 \leqslant \leqslant k \;\; i \leqslant \leqslant 8, \tag{2}$$

where the explicit form of the determinant (3) is given in [4–6].

As it is clear now the problem to be solved reduces to finding of the amplitude of SGEW, determined by Eqs. (1–2), as a function of the incidence angle and polarization of light incident at the CLC surface without TIR.

GENERAL APPROACH TO EXCITATION OF SGEW [2]

To solve the problem of SGEW excitation by a beam incident on a CLC film surface without TIR, one should demand the boundary conditions at this surface and at the surface with TIR to be satisfied for the superposition of free diffracting fields in the film and the SGEW field determined by known coefficients C_j and wave vector q found from the dispersion equation (2).

In general case the free diffracting field in the film is a superposition of four eigen solutions of the diffraction problem (which are known [3]) with coefficients F_j ($j = 1, 4$):

$$E(z) = \sum_{j=1}^{4} F_j e^{-\gamma_{2j} z} [\hat{y}(a_{1j} e^{-i\tau z/2} + a_{3j} e^{i\tau z/2}) + \hat{x}\sin\theta \cdot (a_{2j} \bar{e}^{i\tau z/2} - a_{4j} e^{i\tau z/2}) + \hat{z}\cos\theta (a_{2j} e^{-i\tau z/2} + a_{4j} e^{i\tau z/2})], \tag{3}$$

where the coefficients F_j are to be determined from the boundary conditions.

Finally, the mentioned boundary problem in the general case reduces to the solution of four inhomogeneous linear equations for F_j (or C_j) [2]. For simplification of the problem below the excitation of SGEW is considered in detail for the case of simple SGEW polarization properties compared with the case of the SGEW of first diffraction order [5], namely, for the SGEW of second diffraction order [7].

EXCITATION OF SGEW OF SECOND DIFFRACTION ORDER

In this case the rank of the mentioned above systems of inhomogeneous linear equations is reduced by a half and we need to find only two coefficients F_1 and F_2. The corresponding system for a σ-polarized SGEW of second diffraction order [7] is of the form :

$$(F_1 + C_1)\cos[(\varphi + \beta)/2] + (F_2 + C_2)\cos[(\varphi - \beta)/2] = A/2$$

$$\{(-2\gamma/\tau)\cos[(\varphi + \beta)/2] + \sin[(\varphi + \beta)/2]\}(F_1 + C_1) +$$

$$\{(2\gamma/\tau)\cos[(\varphi - \beta)/2] + \sin[(\varphi - \beta)/2]\}(F_2 + C_2) = -\gamma_1 A/\tau, \qquad (4)$$

where β and γ are determined by the deviation of the incident beam direction (wave-vector æ) from the Bragg condition by the following formulas $\cos\beta = -8\tau(\tau + 2æ)/(\delta æ \cot\theta)^2$, $\gamma = (\delta æ \cot\theta)^2(\sin\beta)/4\tau$ with $æ^2 = \bar{\varepsilon}(\omega/c)^2$.

Connection between the amplitudes of the incident E_i and reflected E_r waves is determined by the following system:

$$(F_1 + C_1)\cos[(\varphi + \beta + \tau L)/2]\exp(\gamma L) + (F_2 + C_2)\cos[(\varphi - \beta + \tau L)/2)]*$$

$$\exp(-\gamma L) = 1/2[\exp(-\gamma_3 L)D + \exp(iæ_n L)E_r + \exp(-iæ_n L)E_i]$$

$$(F_1 + C_1)\exp(\gamma L)\{(-2\gamma/\tau)\cos[(\varphi + \beta + \tau L)/2] + \sin[(\varphi + \beta + \tau L)/2]\}$$

$$+ (F_2 + C_2)\exp(-\gamma L)\{(2\gamma/\tau)\cos[(\varphi - \beta + \tau L)/2] + \sin[(\varphi - \beta + \tau L)/2]\}$$

$$= (\gamma_3/\tau)\exp(-\gamma_3 L)D + i(æ_n/\tau)[\exp(-iæ_n L)E_i - \exp(iæ_n L)E_r], \qquad (5)$$

where $æ_n$ is the normal to the CLC surface component of æ.

CALCULATION RESULTS

Following the formulated above approach the transformation coefficient of the incident light into SGEW $T_r = |A/E_i|$ and the reflection coefficient $R = |E_r/E_i|$ as a function of the beam incidence angle were calculated from (4–5). Results of the corresponding calculations are presented below.

Angular dependence of the transformation coefficient T_r is presented on Figure 2. Analogous dependence of the reflection coefficient R is presented on Figure 3.

Changes of the angular dependence of T_r and R connected with changes of a CLC film thickness are presented on Figure 4 and Figure 5. The presented curves reveal strong dependence of the efficiency of SGEW excitation on the film thickness. An increase of the film thickness results in narrowing of the angular range of effective SGEW excitation and in decrease of the excited SGEW amplitude. The latter finds a quite natural explanation. The diffraction reflection of the incident beam increases with growth of the film thickness and thereby the amplitude of light reaching the surface with TIR decreases which also leads to decrease of the excited SGEW amplitude.

Figure 6 presents angular dependence of the reflection coefficient R for several different propagation directions of excited SGEW. The calculated

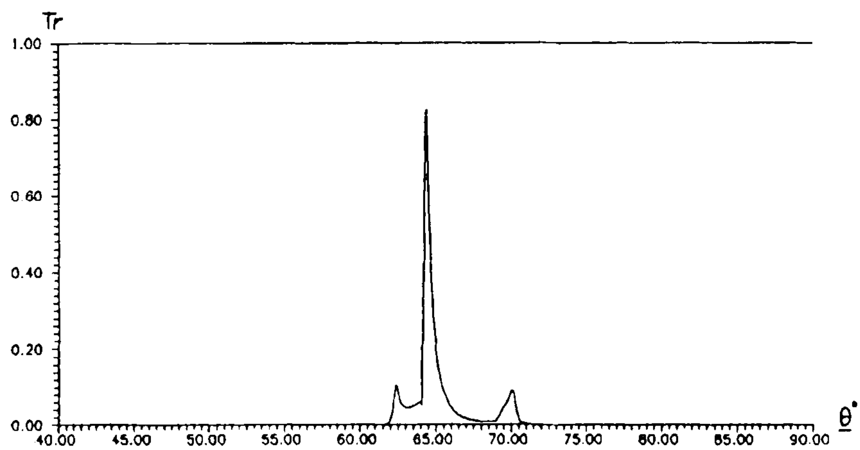

FIGURE 2 Calculated dependence of the transformation coefficient of incident light into SGEW T_r versus the beam incidence angle ($\bar{\varepsilon} = \varepsilon_3 = 1.5$, $\varepsilon_1 = 1$, $\delta = 0.05$, $\phi = 50°$, $L = 40\pi/3\tau$, $\omega/c = \tau$).

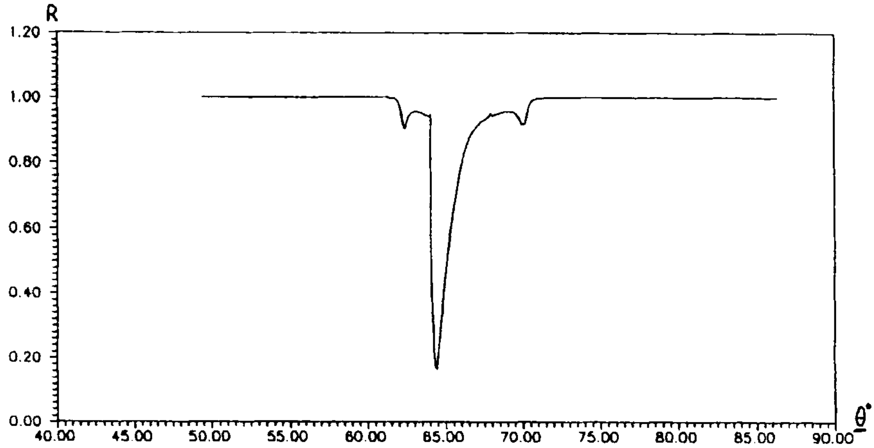

FIGURE 3 Calculated dependence of the amplitude reflection coefficient of incident light R versus the beam incidence angle ($\bar{\varepsilon} = \varepsilon_3 = 1.5$, $\varepsilon_1 = 1$, $\delta = 0.05$, $\phi = 50°$, $L = 40\pi/3\tau$, $\omega/c = \tau$).

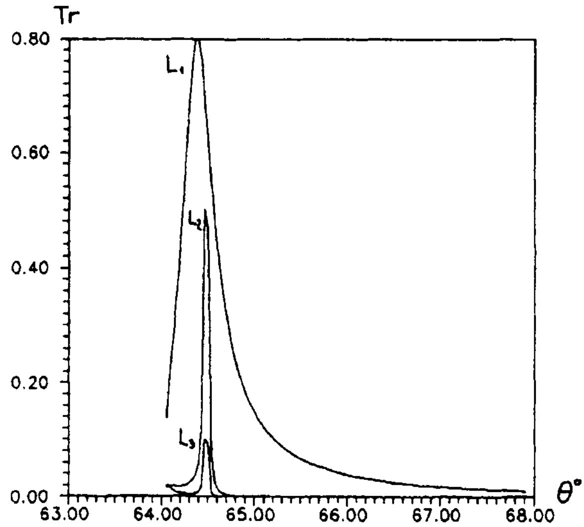

FIGURE 4 Calculated dependence of the transformation coefficient of incident light into SGEW T_r versus the beam incidence angle in the range of strong diffraction reflection for three different CLC film thicknesses ($\bar{\varepsilon} = \varepsilon_3 = 1.5$, $\varepsilon_1 = 1$, $\delta = 0.05$, $\phi = 50°$, $L_1 = 40\pi/3\tau$, $L_2 = 148\pi/3\tau$, $L_3 = 208\pi/3\tau$, $\omega/c = \tau$).

curves reveal quite essential dependence of the reflection coefficient and consequently of the transformation coefficient on the direction of SGEW propagation.

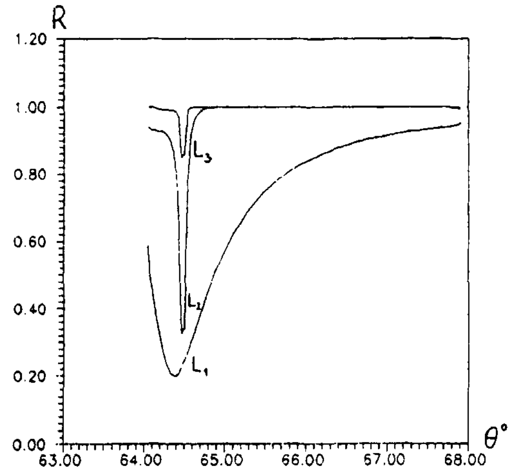

FIGURE 5 Calculated dependence of the amplitude reflection coefficient R versus the beam incidence angle in the range of strong diffraction reflection for three different CLC film thicknesses ($\bar{\varepsilon} = \varepsilon_3 = 1.5$, $\varepsilon_1 = 1$, $\delta = 0.05$, $\phi = 50°$, $L_1 = 40\pi/3\tau$, $L_2 = 148\pi/3\tau$, $L_3 = 208\pi/3\tau$, $\omega/c = \tau$).

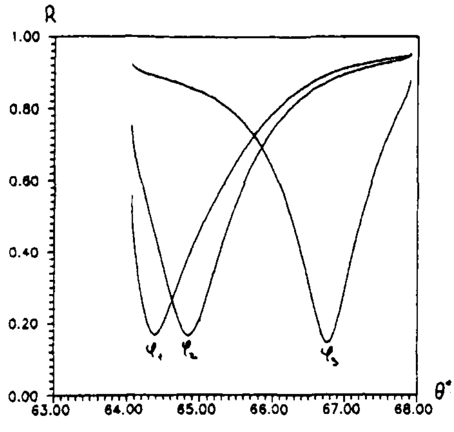

FIGURE 6 Calculated dependence of the amplitude reflection coefficient R versus the beam incidence angle in the range of strong diffraction reflection for three different propagation directions of SGEW relative to the director orientation on CLC interface with TIR ($\bar{\varepsilon} = \varepsilon_3 = 1.5$, $\varepsilon_1 = 1$, $\delta = 0.05$, $L = 40\pi/3\tau$, $\omega/c = \tau$, $\phi_1 = 50°$, $\phi_2 = 30°$, $\phi_3 = -30°$).

CONCLUSION

The presented above results of calculations of the SGEW excitation in a CLC films relate to the case of simplest SGEW polarization properties. However, these results may be used as a qualitative guide for more complicated cases because they reveal some general features of excitation SGEW in CLC films.

In general, the performed investigation demonstrates an essential dependence of the conditions of efficient SGEW excitation on the CLC film thickness and on the SGEW propagation direction relative to the director orientation on the interface with TIR. In particular, sharp dependence of the angular interval of incidence angles corresponding to efficient SGEW excitation on the CLC film thickness, the thicker film the narrower interval, is a general property which is also coupled with decreasing of the transformation coefficient T_r with increase of the film thickness.

Note, that the proposed here method of SGEW excitation by a beam incident at the interface without TIR was not realized before. Mention, that just the fact of existence of a dip in the angular dependence of reflection coefficient can be considered as a proof of the SGEW existence in the CLC structures under investigation.

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